Laurent Moonens (Orsay, France).

Differentiation along rectangles

Lebesgue's differentiation theorem states that, when f is a locally integrable function in Euclidean space, its average on the ball $B(x,r)$ centered at x with radius r, converges to $f(x)$ for almost every x, when r approaches zero. Many questions arise when the family of balls $\{B(x,r)\}$ is replaced by a *differentiation basis* $\mathcal{B} = \bigcup_x \mathcal{B}_x$ (where, for each x, $\mathcal{B}_x$ is, roughly speaking, a collection of sets shrinking to the point x). In this case, one looks for conditions on $\mathcal{B}$ such that the average of f on sets belonging to $\mathcal{B}_x$ are known to converge to $f(x)$ for a.e. x, when those sets shrink to the point x. Many interesting phenomena happen when sets in $\mathcal{B}$ have a *rectangular* shape (Lebesgue's theorem may or may not hold in this case, depending on the geometrical properties of sets in $\mathcal{B}$). In this talk, we shall review some of the history around this problem, as well as recent results obtained with E. D'Aniello and J. Rosenblatt.