Introduction to linear and nonlinear theory of capacities

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Abstract

In this talk, we present an introduction of the classical linear capacity theory through Riesz potentials I_α that has its origin in physics. Physical considerations were taken into account in order to choose the Riesz kernel. The talk has ground in H. Cartan's ideas for linear capacity theory, where he adds a Hilbert space structure on the set of measures μ with finite $\alpha-$energy integral

$$\|\mu\|^2 = E_\alpha(\mu) := \iint K_\alpha(x-y)d\mu(x)d\mu(y).$$

Thus, the capacity $C_{\alpha,2}(K)$ of the compact set K arises from a minimizing sequence of nonnegative measures $\{\mu_n\}$ with finite $\alpha-$energy integral. This capacity has nice properties similar to the measure theory, however the capacity is not additive. We finish the talk with existence of $C_{\alpha,2}(E)-$extremal measures and indicate how to introduce explicitly a nonlinear capacity theory as proposed by Mazya in a sequence of papers from 1960 to 1970.

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