Improved error estimate for the order of strong convergence of the Euler method for random ordinary differential equations

Ricardo M. S. Rosa[1] and Peter Kloeden[2]

[1] *Universidade Federal do Rio de Janeiro, Brasil*
[2] *University of Tübingen, Germany*

The Euler method for approximating an ODE is known to be of order 1. For Stochastic ODEs with multiplicative noise, however, it drops to $\frac{1}{2}$. What about for Random ODEs? Current works tell us it is also order $\frac{1}{2}$, or even less, depending on the Holder exponent of the noise sample paths. Here we show that in many typical situations, it is actually of order 1. This applies to a variety of noises, such as additive or multiplicative Itô processes, transport processes with sample paths of bounded variation, and even processes with discontinuous sample paths, as in point-process noises. For fractional Brownian motion processes, we may not reach order 1, depending on the Hurst parameter, but we still improve the order compared with the current belief. The proofs rely on writing a global error formula instead of estimating the local error; using Fubini to move the critical regularity term to the larger scales; and using the Itô isometry or some other form of global estimate to control that critical term. In this talk, we discuss these improvements, sketch the proofs, and illustrate the results numerically with a number of interesting models. This is a joint work with Peter Kloeden (University of Tübingen, Germany).