From stochastic hamiltonian systems to stochastic compressible Euler equation

Jesus Manuel Correa Lora and Christian Horacio Olivera

Universidade de Campinas (UNICAMP), Brasil

For all $N \in \mathbb{N}$ we consider N particles in $\mathbb{R}^d$ where the position $X_t^{k,N}$ verifies for $k = 1, \cdots, N$

$$d^2 X_t^{k,N} = -\frac{1}{N} \sum_{l=1}^{N} \nabla \phi_N \left(X_t^{k,N} - X_t^{l,N} \right) dt + \sigma \left(X_t^{k,N} \right) \frac{dX_t^{k,N}}{dt} \circ dB_t \qquad (1)$$

where $\{B_t^i)_{t \in [0,T]},\ \ i \in \mathbb{N}\}$ is a family of standard $\mathbb{R}^d$-valued Brownian motions defined on a filtered probability space. Our aim is the study of the asymptotics as $N \to \infty$ of the time evolution of the whole system of all particles. Therefore, we investigate the empirical processes:

$$S_t^N : \ = \frac{1}{N} \sum_{k=1}^{N} \delta_{X_t^{k,N}},$$

$$V_t^N : \ = \frac{1}{N} \sum_{k=1}^{N} V_t^{k,N} \delta_{X_t^{k,N}}$$

where $dX_t^{k,N} : \ = V_t^{k,N} dt$ is the velocity of the kth particle, and δ_a, denotes the Dirae measure at a. We shall prove that S_t^N and V_t^N converge as $N \to \infty$ to solutions of the continuity equation and the stochastic Euler equation, respectively.