

Smoothness of densities for path-dependent SDEs under Hörmander's condition

Evelina Shamarova¹, Alberto Ohashi² and Francesco Russo¹

¹*Universidade Federal da Paraíba, Brasil*

²*Universidade de Brasília, Brasil*

³*ENSTA Paris, Institut Polytechnique de Paris, Unite de Mathématiques appliquées, France*

We establish the existence of smooth densities for solutions to a broad class of path-dependent SDEs under a Hörmander-type condition. The classical scheme based on the reduced Malliavin matrix turns out to be unavailable in the path-dependent context. We approach the problem by lifting the given n -dimensional path-dependent SDE into a suitable L_p -type Banach space in such a way that the lifted Banach-space-valued equation becomes a state-dependent reformulation of the original SDE. We then formulate Hörmander's bracket condition in \mathbb{R}^n for non-anticipative SDE coefficients defining the Lie brackets in terms of vertical derivatives in the sense of the functional Itô calculus. Our pathway to the main result engages an interplay between the analysis of SDEs in Banach spaces, Malliavin calculus, and rough path techniques.