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Weighted Inequalities for the Fractional Laplacian and the Existence of Extremals

We obtain improved versions of Stein-Weiss [1] and Caffarelli-Kohn-Nirenberg inequalities [3], involving Besov norms of negative smoothness.

This kind of inequalities were originally obtained in the unweighted case in P. Gerard, Y. Meyer, and F. Oru in [4] by using Littlewood-Paley theory. Instead, we use a simpler technique inspired by an argument of D. Chamorro in [2] which uses only the Stein-Weiss inequality and the boundedness of the Hardy-Littlewood maximal function with A_p weights.

As an application of the former, we derive the existence of extremals of the Stein-Weiss inequality in certain cases, some of which are not contained in the celebrated theorem of E. Lieb [5].

This is a joint work with Irene Drelichman and Ariel Salort.

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