Book of Abstracts
Welcome

The Department of Computation and Mathematics of the Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto of the Universidade de São Paulo (DCM-FFCLRP-USP) and the GAFEVOL group welcome you to the IX Conference GAFEVOL which will take place at the city of Ribeirão Preto, in the state of São Paulo, Brazil, on September 16-18, 2014.

Organizing committee

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Address

IX Conference GAFEVOL
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1. Department of Computation and Mathematics (DCM)
2. Didactic Building
3. Canteen
4. Parking
General Information
Conference site

The meeting will take place at Auditorium (Room DE-11) and also in the Coffee Area. The Auditorium is at Didactic Building and the Coffee Area is at DCM. See the map on page 11.

Registration

The registrations will be made in the following schedule:

- **Monday, September 15th**: From 14:00hs to 14:40hs in the lobby of Pousada Santa Rita.
- **Monday, September 15th**: From 17:10hs to 17:50hs in the lobby of Pousada Santa Rita.
- **Tuesday, September 16th**: From 8:00hs to 8:50hs in the DCM.

We will provide you a badge at registration. Please wear your badge at the event.

Financial support

The financial support payment is planned to be made on Wednesday, September 17th.

Social events

- **Tuesday, September 16th**: Photo of the meeting at 17:10 at DCM.
- **Tuesday, September 16th**: Music presentation at 17:50 at DCM.
- **Tuesday, September 16th**: Cocktail at 19:00 at DCM.
- **Wednesday, September 17th**: Conference Dinner at 20:30 at the Churrascaria Estância

Computer and wireless LAN use

There will be available computers for use at Informatics Laboratory (room 601 at DCM). Also, all the participants of the IX Conference GAFEVOL can use this laboratory during the days of the conference from 8:00 a.m. to 18:00 p.m.

In order to access the wireless connection at the University you need to follow the steps:

1. Enable wireless on your device.
2. Join the USP-NET wireless network.
3. Open a browser and attempt to visit a website (for example your home page).
4. Click on the button in the page to proceed.

5. You will be redirected to a login page. Enter the login and password which you will receive at the registration day.

6. You may freely browse the internet after logging in. You may occasionally need to re-authenticate using the above procedure.

**Meals and refreshments**

There is a canteen available at DCM where you can have snacks. Also, all the lunches during the conference will be there. See the map on page 11.

Also, there are several restaurants and bars in the city. Some of them are:

- **Pinguim Bar and Restaurant (regional beer and restaurant house)**
  
  **Address:** Street General Osório, 389, Centro, Ribeirão Preto
  
  **Contact Number:** (16) 3610-8258
  
  **Website:** http://www.pinguimochopp.com.br/

- **Colorado Cervejarium (Regional beer house)**
  
  **Address:** Av. Independência, 3.242, Ribeirão Preto
  
  **Contact Number:** (16) 3911.4949
  
  **Website:** http://www.coloradocervejarium.com.br/

- **Nelson Restaurant and Bar**
  
  **Address:** Street Prudente de Morais, 1313, Centro, Ribeirão Preto
  
  **Contact Number:** (16) 3625-6669
  
  **Website:** http://www.bardonelsonrp.com.br/

- **Churrascaria Estância**
  
  **Address:** Av. Presidente Vargas, 1100, Alto da Boa Vista, Ribeirão Preto
  
  **Contact Number:** (16) 3911-9513
  
  **Website:** http://www.estanciaribeirao.com.br/

The Conference Dinner will be at the Churrascaria Estância.

**Health emergencies**

In case of accidents or health emergencies call 192 (SAMU).
Money exchanges

In case you need to exchange your money, we recommend you to look for the following agencies:

- **Confidence Cambio Exchange**
  - **Address 1:** Av. Coronel Fernando Ferreira Leite, 1540, Jd. Califórnia, Ribeirão Shopping
  - **Address 2:** Street São José, 933, Shopping Santa Úrsula.
  - **Contact Number:** 4004 5700
  - **Website:** [http://www.confidencecambio.com.br/](http://www.confidencecambio.com.br/)
  - **Open hours:** Monday to Friday from 10:00 a.m. to 20:00 p.m. and Saturday from 10:00 a.m. to 16:00 p.m.

- **Daycoval Cambio**
  - **Address:** Av. Presidente Vargas, 1617
  - **Contact Number:** (16) 3620 2043 / 3621 0512
  - **Website:** [http://www.daycoval.com.br/](http://www.daycoval.com.br/)
  - **Open Hours:** Monday to Friday from 10:00 a.m. to 19:00 p.m

Taxis

In case you need to use a taxi, we recommend the following agencies:

- **Coopertaxi**
  - **Contact number:** (16) 3323-7000

- **Aliança Rádio Táxi**
  - **Contact number:** (16) 3911-3000

Tourism

We recommend some nice places to visit during your stay in Ribeirão Preto.

- **Parks**
  1. **Curupira Park (Park Prefeito Luiz Roberto Jâbalí)**
     - **Address:** Av. Costabile Romano, 337, Ribeirão Preto-SP
  2. **Municipal Park Raia (Municipal Park Dr. Luiz Carlos Raia)**
     - **Address:** Street Severino Amaro dos Santos, Ribeirão Preto-SP
  3. **Bosque and Zoo Fábio Barreto**
     - **Address:** Street Liberdade s/n, Ribeirão Preto, Estado de São Paulo, Brasil
     - **Contact Number:** (16) 3636-2545 / 3636-2283
     - **Website:** [http://www.ribeiraoopreto.sp.gov.br/turismo/zoologico/i71principal.php](http://www.ribeiraoopreto.sp.gov.br/turismo/zoologico/i71principal.php)
• Theater and Museum

1. **Theater Pedro II**
   - **Address:** Street Álvares Cabral, 370, Ribeirão Preto-SP
   - **Contact Number:** (16) 3977 8111
   - **Website:** [http://www.theatropedro2.com.br/](http://www.theatropedro2.com.br/)

2. **Coffee Museum Francis Schmidt**
   - **Address:** Av. Zeferino Vaz, s/n Campus da USP, Monte Alegre, Ribeirão Preto-SP
   - **Contact Number:** (16) 3633-1986

**Smoking**

Smoking is prohibited in any of the DCM buildings.
Programme
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Abstracts
Semilinear fractional differential equations with critical nonlinearities

Bruno de Andrade

Abstract

In this work we study existence of regular mild solutions to abstract fractional Cauchy problems of order $\alpha \in (0,1)$ with critical nonlinearities. Concretely, we analyze the existence of local regular mild solutions for the problem, and its possible continuation to a maximal interval of existence. We apply our abstract results to fractional partial differential equations coming from heat conduction theory.

This is joint work with Alexandre Nolasco de Carvalho (ICMC-USP), Paulo Carvalho-Neto (IMECC-UNICAMP) and Pedro Marín-Rubio (EDAN-US).

Bibliography

Controllability for systems of neutral type with delay

Fernando Gomes de Andrade* and Andréa Cristina Prokopczyk Arita

Abstract

This work is a study of the approximate controllability of a system of neutral type described by

\[
\frac{d}{dt} \left( x(t) + F(t)(x_t) \right) = Ax(t) + L(t)(x_t) + Bu(t), \quad t \geq 0, \tag{0.0.1}
\]

\[
x_0 = \varphi \in \mathcal{C}, \tag{0.0.2}
\]

where \( x(t) \in X, u(t) \in U \), for all \( t \geq 0 \), \( X \) is the state space, \( U \) is the space of control, both are Hilbert spaces, \( \mathcal{C} = C([-r, 0], X) \) is the space of continuous functions from \([-r, 0]\) to \( X \), for each \( t \), \( x_t : [-r, 0] \to X \) is the history of \( x \) at \( t \), i.e., \( x_t(\theta) = x(t+\theta) \), for all \( \theta \in [-r, 0] \). \( L : [0, \infty) \to \mathcal{L}(\mathcal{C}, X) \) is strongly continuous, i.e., \( t \mapsto L(t)\psi \) is continuous for all \( \psi \in \mathcal{C} \) fixed, furthermore, \( A : D(A) \subset X \to X \) is the infinitesimal generator of analytic semigroup on \( X \), \( B : U \to X^\beta \) is a bounded linear operator, \( \beta \in (\frac{1}{2}, 1) \), \( X^\beta = D((-A)^\beta), \| \cdot \|_\beta \), for some \( \beta \in (\frac{1}{2}, 1) \), is a Banach space with \( \| x \|_\beta = \| ((-A)^\beta x) \|, \forall x \in D((-A)^\beta) \), and \( F : [0, +\infty) \to \mathcal{L}(\mathcal{C}, X^\beta) \), is strongly continuous and satisfies the following Lipschitz condition

\[
\| ((-A)^\beta F(t)(\psi_1) - (-A)^\beta F(t)(\psi_2)) \| \leq C_0(|t - s| + \| \psi_1 + \psi_2 \|_C),
\]

for all \( t, s \in [0, \tau], \psi_1, \psi_2 \in \mathcal{C} \) and some constant \( C_0 > 0 \).

Our aim is to use the ideas presented in [1] and [2] to compare the controllability of the linear system without delay

\[
x'(t) = Ax(t) + Bu(t), \quad t \geq 0, \tag{0.0.3}
\]

\[
x(0) = x^0 \in X. \tag{0.0.4}
\]

with the controllability of neutral system with delay (0.0.1)-(0.0.2).

Bibliography


Partially supported by CAPES, Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, São Carlos, Brasil, e-mail: andrade_fg@usp.br
Bifurcation of Periodic Solutions for Retarded Functional Differential Equations on Manifolds

Pierluigi Benevieri*, Alessandro Calamai, Massimo Furi and Maria Patrizia Pera

Abstract

We consider $T$-periodic parametrized retarded functional differential equations, with infinite delay, on (possibly) noncompact manifolds. Using a topological approach, based on the notions of degree of a tangent vector field and of the fixed point index, we prove a global continuation result for $T$-periodic solutions of such equations. As corollaries we obtain a Rabinowitz type global bifurcation result and a continuation principle of Mawhin type.

Bibliography


Department of Mathematics, University of São Paulo, São Paulo, Brazil
On the Jack Hale's problem for impulsive systems

Everaldo de Mello Bonotto

Abstract

In this work, we study the Jack Hale’s problem for impulsive dynamical systems. In order to solve the problem of Jack Hale, we use the concept of asymptotic compactness for impulsive systems.

Bibliography


Existence of solutions for abstract neutral nonlinear fractional
differential degenerate equations

Eduardo Hernández, Alejandro Caicedo Roque* and Donal O’Regan

Abstract

In this work we continue the developments in [1] on abstract fractional differential equations. Specifically, we study the existence and qualitative properties of solutions for a class of abstract degenerate neutral differential equations with fractional temporal derivative of the form

\[ D_0^\alpha (Lx(t) + g(t, x_t)) = Ax(t) + f(t, x_t), \quad t \in [0, a], \tag{0.0.5} \]
\[ x_0 = \varphi \in B = C([-p, 0]; X), \tag{0.0.6} \]

where \( 0 < \alpha < 1 \), \( D_0^\alpha \) denotes the \( \alpha \)-fractional derivative in the Caputo sense, \( A : D(A) \subset X \to X \) is a sectorial operator, \((X, \| \cdot \|)\) is a Banach space, the history \( x_t \) belongs to \( C([-p, 0]; X) \) and \( f(\cdot), g(\cdot) \) are suitable continuous functions.

Bibliography


Pullback attractor and extremal complete trajectories

Érika Capelato* and Ricardo de Sá Teles

Abstract

The subject of this paper is to analyze the asymptotic behavior of the nonlinear nonautonomous problem

$$
\begin{cases}
    u_t - \text{div}(|u|^{p(x)-2}\nabla u) = B(t, u) \\
    u(\tau) = u_0 \in L^2(\Omega),
\end{cases}
$$

(0.0.7)

where $\Omega$ is a bounded smooth domain in $\mathbb{R}^n$, $n \geq 1$, $p(x) \in C(\Omega)$, $2 + \delta \leq p(x) \leq 3 - \delta$, $\delta > 0$ a.e. $x \in \Omega$. We will suppose that $B : \mathbb{R} \times L^2(\Omega) \to L^2(\Omega)$ is globally Lipschitz and increasing. In the Banach space, $W^{1,p(x)}(\Omega)$, defined by [1] we obtain an estimate for the solution of the problem (0.0.7) using results of the [3] and thus, we have proved the existence of the pullback attractor for this problem. To prove the existence of the extremal complete trajectories, that “delimitate” in a certain sense the pullback attractor, we observe that principal part of the problem is a maximal monotone operator and can also be seen as the subdifferential of a lower semicontinuous convex function (see [2]) and furthermore, the process is monotonous.

Bibliography


An inverse problem in biological olfactory cilium

Carlos Conca

Abstract

In this lecture we study a linear inverse problem with a biological interpretation, modelled by a Fredholm integral equation of the first kind, where the kernel is represented by step functions. Based on different assumptions, identifiability, stability and reconstruction results are obtained.
Well-posedness and qualitative aspects of solutions with datum on Besov-Morrey spaces for a diffusion-wave equation

Marcelo Fernandes de Almeida

Abstract

This paper concerns with an interpolated Parabolic-Hyperbolic PDE arisen of time-fractional integrodifferential equations. Global existence in critical Besov-Morrey spaces $\mathcal{L}_{p,\mu,\infty}^{s}(\mathbb{R}^n)$ ($n \geq 1$) and qualitative aspects, like symmetries and positivity of solutions, is showed. Moreover, asymptotic behavior of solutions is proved in the framework of scaling invariant Besov-Morrey spaces. Also, self-similarity of solutions is investigated.

Bibliography


Federal University of Sergipe, Department of Mathematics, Aracaju, Brazil, e-mail: nucaltiado@gmail.com
Cycles by interaction of damping and jumps of energy

Miguel V. S. Frasson*, Marta C. Gadotti, Selma H. J. Nicola and Plácido Z. Táboas

Abstract

We consider a linear oscillator with damping combined with an autonomous impulsive condition inspired by Myshkis [3]. We adopt a different approach to extend some of his results and prove that there are asymptotically orbitally stable cycles and stable orbits arising from period doubling bifurcations.

Consider the discontinuous dynamical system on the plane $\dot{x}$ arising from the damped linear oscillator

$$\ddot{x} + 2\alpha \dot{x} + \omega^2 x = 0,$$

and suppose that when the total energy reaches some critical level, the velocity undergoes a fixed instantaneous increase. Without loss of generality we may consider the impulsive system

$$\begin{align*}
  \dot{x} &= y, \\
  \dot{y} &= -x - 2ay, \\
  x^2(t) + y^2(t) &= 1 \quad \Rightarrow \quad (x(t+), y(t+)) = (x(t), y(t) + v)
\end{align*}$$

where $0 < a < 1$ and $v > 0$ are parameters. The orbits of the linear system of ordinary equations spirals clockwise about the origin with forward $t$, with $(x(t), y(t)) \to 0$ as $t \to \infty$ and $|(x(t), y(t))| \to \infty$ as $t \to -\infty$. The impulse condition depends only in the current state. We have therefore a autonomous impulsive system.

Periodic solutions of (0.0.8) are called cycles and were studied in [3]. A simple cycle has just one instant of impulse within a minimal period. A simple cycle $z(t) = (x(t), y(t))$ is positive if $x(t) > 0$, $\forall t$. The value $\beta$ over which the impulse occurs is called a vertex. A global solution $u$ of (0.0.8) is orbitally stable, (resp. orbitally asymptotically stable or orbitally unstable) if its orbit $\gamma = \{u(t) \mid -\infty < t < \infty\}$ is stable (resp. asymptotically stable or unstable) as a set. We identify $S^1$ with the real line though the usual parametrization $\theta \mapsto e^{i\theta}$ without further comments.

Results

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Proposition 0.1 (Existence of simple cycles) Fix $0 < a < 1$. For each $\beta \in [-2\pi, 0)$ there exists a sequence $(v_n)$, $n \geq 0$ with $v_n > 0$ such that (0.0.8) with $v = v_n$ has a simple cycle $z(t)$ with vertex $\beta = z(0)$. The simple cycles are distinguished by their indexes. Let $v_{*\beta} = v_0$. Furthermore if $-3\pi/2 < \beta < 0$, (respectively if $-2\pi \leq \beta \leq -3\pi/2$) then we have cycle have minimal indexes for $v = v_{*\beta}$, that is, $\ell = 0$ (respectively $\ell = 1$).

Theorem 0.2 A cycle $\gamma$ of (0.0.8) with vertex $\beta \in [-\pi, 0)$ given by Proposition 0.1 is orbitally unstable for $|\beta|$ sufficiently small and orbitally asymptotically stable for $\beta$ in a neighborhood of $-\pi/2$.

Theorem 0.3 For $a > 0$ sufficiently small, there exists $\beta^*$ such that for $\beta < \beta^*$ we have that the simple cycle of (0.0.8) with $v = v_{*\beta}$ is orbitally stable and as $v$ crosses the bifurcation value $v_{*\beta}$, the cycles undergo a period doubling bifurcation, that is, a branch of orbitally stable non-simple cycles arises with the period of these cycles tending to the double of the simple cycle of $v = v_{*\beta}$ as $v \to v_{*\beta}^+$.

Bibliography


Dominant Solutions of Delay Differential Equations

István Györi

Abstract

In this talk, we describe several situations in which all solutions of a delay differential equation can asymptotically be characterized by appropriate "dominant" solutions. The class of equations include linear autonomous equations, quasilinear equations and nonautonomous equations with small delays. This is a joint work with my colleagues Ferenc Hartung and Mihály Pituk (University of Pannonia, Hungary).

Bibliography


Second Order Abstract Neutral Functional Differential Equations

Hernán R. Henríquez* and Claudio Cuevas

Abstract

In this paper we are concerned with a class of second order abstract neutral functional differential equations with finite delay in a Banach space. We establish the existence of mild and classical solutions for the nonlinear equation, and we show that the map defined by the mild solutions of the linear equation is a strongly continuous semigroup of bounded linear operators on an appropriate space. We use this semigroup to establish a variation of constants formula to solve the inhomogeneous linear equation.

Introduction

The aim of this work is to develop the basic theory for a class of second-order abstract neutral functional differential equations. In this work, $X$ denotes a Banach space endowed with a norm $\| \cdot \|$. Moreover, for a fixed constant $r > 0$ that represents the delay of the equation, we denote by $C([-r,0],X)$ the space of continuous functions from $[-r,0]$ into $X$ provided with the norm of uniform convergence. We are concerned with the the semilinear abstract Cauchy problem

$$\frac{d^2}{dt^2} D(x_t) = AD(x_t) + f(t,x_t,x'_t), \quad t \geq 0,$$

(0.0.9)

$$x_0 = \varphi^0, \quad x'_0 = \psi^0.$$

(0.0.10)

where $x(t) \in X$, the function $x_t : [-r,0] \to X$, that denotes the segment of $x(\cdot)$ at $t$, is given by $x_t(\theta) = x(t+\theta)$. We assume that $D : C([-r,0],X) \to X$ is a bounded linear map, and $f$ is an appropriate $X$-valued function.

Results

To study problem (0.0.9)-(0.0.10) we assume that $A : D(A) \subseteq X \to X$ is the infinitesimal generator of a cosine family of bounded linear operators $(C(t))_{t \in \mathbb{R}}$ on $X$, with associated sine function $S(t)$, $D : C([-r,0],X) \to X$ is a bounded linear map, and $f : I \times C([-r,0],X) \times C([-r,0],X) \to X$ is a function that satisfies the Carathéodory conditions. We assume that

$$D(\varphi) = \varphi(0) - P(\varphi), \quad \varphi \in C([-r,0],X),$$

Partially supported by CONICYT, under Grant FONDECYT 1130144 and DICYT-USACH, Department of Mathematics, University of Santiago-USACH, Santiago, Chile, e-mail: hernan.henriquez@usach.cl
where $P : C([-r, 0], X) \to X$ is a bounded linear map defined by
\[ P(\varphi) = \int_{-r}^{0} [d\mu(\theta)] \varphi(\theta), \quad \varphi \in C([-r, 0], X), \]
where $\mu : [-r, 0] \to \mathcal{L}(X)$ is a map of bounded variation and non-atomic at zero.

In this general framework, and assuming that the function $f$ has different properties, we show that the problem (0.0.9)-(0.0.10) admits a solution. We distinguish between mild solutions and classical solutions. In particular, we establish the linear equation
\[
\frac{d^2}{dt^2} D(x_t) = A D(x_t) + \Lambda_1(x_t) + \Lambda_2(x_t') + h(t), \quad t \geq 0, \tag{0.0.11}
\]
with initial condition (0.0.10), where $\Lambda_1, \Lambda_2 : C([-r, 0], X) \to X$ are bounded linear maps, and $h : [0, \infty) \to X$ is a locally integrable function, $\varphi^0 \in C^1$, $\psi^0 = \frac{d}{d\theta} \varphi^0$ and $D(\varphi^0) \in E$, admits a unique solution. We introduce the space
\[
C_D^1 = \{ \varphi \in C^1([-r, 0], X) : D(\varphi) \in E \}
\]
provided with the norm
\[
|||\varphi||| = ||D(\varphi)||_E + ||\varphi||_{\infty} + ||\varphi'||_{\infty}, \quad \varphi \in C_D^1,
\]
where $E$ is the Kisyński space. Initially we consider the homogeneous case, i.e. we take $h(t) = 0$ for $t \geq 0$. Let $\varphi^0 \in C_D^1$ and $x(\cdot, \varphi^0)$ be the mild solution of problem (0.0.11)-(0.0.10). We define the map
\[
U(t) \varphi^0 = x_t(\cdot, \varphi^0), \quad t \geq 0.
\]

**Theorem 0.4** Under the above conditions, the family $(U(t))_{t \geq 0}$ is a strongly continuous semigroup of bounded linear operators on $C_D^1$.

Using the semigroup $U(t)$ we establish a type of variation of constant formula to solve the nonhomogeneous problem (0.0.11)-(0.0.10). We define the bounded linear operator $V(t) : C_D^1 \to C([-r, 0], X)$ by
\[
V(t) \varphi^0 = v_t, \quad t \geq 0,
\]
where $v(t) = u'(t)$ and $u_t = U(t) \varphi^0$.

**Corollary 0.5** Under the above conditions, let $\varphi^0 \in C_D^1$ and $\psi^0 = \frac{d}{d\theta} \varphi^0$. If $u(\cdot)$ is the mild solution of problem (0.0.11)-(0.0.10), and $v = u'$, then
\[
u_t = V(t) \psi^0 + \lim_{\lambda \to \infty} \int_0^t U(t-s) \Lambda_1(\varphi^0) \Delta(\lambda) \psi^0 ds,
\]
\[
u_t = V(t) \psi^0 + \lim_{\lambda \to \infty} \int_0^t V(t-s) \Lambda_1(\varphi^0) \Delta(\lambda) \psi^0 ds.
\]

For a proof of these results, we refer the reader to [1].
Bibliography

A unified approach to discrete fractional calculus and applications

Sebastián Calzadillas, Carlos Lizama* and Jaqueline G. Mesquita

Abstract
We present a unified treatment of several existing definitions of discrete fractional sums and differences by means of the use of the operator of translation. We then formulate a standard notion of fractional sum by finite convolution, and we state their main properties in the space of vector-valued sequences $s(N_0; X)$ where $X$ is a Banach space. We introduce the notion of generalized Mittag-Leffler sequence by means of the complex inversion of the $Z$-transform, and use it to solve, for $\lambda \in \mathbb{C}$ the non-homogeneous problem

$$\Delta^\alpha u(n) = \lambda u(n) + f(n)$$

where $f \in s(N_0; X)$ and $0 < \alpha \leq 2$. Here, the fractional difference is defined both as the discrete analogous to the Caputo fractional derivative as well as the Riemann-Liouville. We recover, improve and extend several notions and applications in the existing literature on the subject.

Keywords: Fractional sums; fractional differences; $Z$-transform; fractional difference equations; convolution; translation

MSC 2010 subject classification: 39A13; 34A08; 44A15; 44A35

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Almost automorphic solutions for evolutions equations

Arlucio Viana, Bruno de Andrade and Eder Mateus*

Abstract

In recent years, the theory of almost automorphic functions has been developed extensively and consequently there has been a considerable interest in the existence of almost automorphic solutions of various kinds of evolution equations, see for instance [4, 6, 8, 10] and the references therein. In this work, we study existence and uniqueness of almost automorphic mild solutions for a class of abstract differential equations described in the form

\[ u'(t) = Au(t) + f(t, u(t)), \quad t \in \mathbb{R}, \]  

(0.0.12)

where \( A \) is an unbounded linear operator, assumed to be Hille-Yosida of negative type, with domain \( D(A) \) not necessarily dense on some Banach space \( X \). \( f : \mathbb{R} \times X_0 \to X \) is a continuous function and \( X_0 = D(A) \). We ensure sufficient conditions for existence and uniqueness of almost automorphic solutions to (0.0.12) with Stepanov almost automorphic conditions. We apply our abstract results in the framework of transmission problems for the Bernoulli-Euler plate equation and heat conduction theory.

Bibliography


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Continuous solutions for divergence-type PDEs

Laurent Moonens

Abstract

It follows from a famous work by J. Bourgain and H. Brezis [1] that given $F \in L^n$ whose integral vanishes on the $n$-dimensional torus, there exists a continuous vector field $v$ satisfying $\text{div} \, v = F$. We shall review some necessary and sufficient conditions (obtained by T. De Pauw and W.F. Pfeffer [2]) on a distribution $F$ in order that the equation $\text{div} \, v = F$ has a continuous solution, and provide a similar result on the existence of continuous solutions to the equation $d\omega = F$ in the class of $m$-differential forms (the latter part being a joint work with T. De Pauw and W.F. Pfeffer [3]).

Bibliography


Partially supported by ANR Project “Geometrya”, Laboratoire de Mathématique, Université Paris-Sud, 91405 Orsay Cedex, France, e-mail: laurent.moonens@math.u-psud.fr
Dengue: Model with human mobility

S. Oliva

Abstract

We present recent models dealing with the spread of infectious diseases, we start with the simple SIS model and build it up to include human mobility. The understanding of human mobility and the development of qualitative and quantitative models are key to understand human infectious diseases. We fix, as an example, Dengue. We try to connect several dynamic models, from systems of ordinary differential equations to evolution equations with fractional powers of the Laplacian.
Dynamics of parabolic equations governed by the p-laplacian on unbounded thin domains

Ricardo Parreira da Silva

Abstract

We consider the asymptotic behavior of quasilinear parabolic equations posed in a family of unbounded domains that degenerates onto a lower dimensional set. Considering an auxiliary family of weighted Sobolev spaces we show the existence of global attractors and we analyze convergence properties of the solutions as well of the attractors.

Bibliography


Thin domains and reactions concentrated on boundary

Marcone C. Pereira

Abstract

In this talk we discuss the behavior of a family of steady state solutions of a semilinear reaction-diffusion equation with homogeneous Neumann boundary condition posed in a two-dimensional thin domain when some reaction terms of the problem are concentrated in a narrow oscillating neighborhood of the boundary. We assume that the domain, and so, the oscillating neighborhood, degenerates to an interval as a small parameter $\varepsilon$ goes to zero.

Our main goal here is to show that this family of solutions converges to the solutions of an one-dimensional limit equation capturing the geometry and oscillatory behavior of the open sets where the problem is established.

Indeed, we introduce a model combining these both singular situations in a more general featuring. Here we adapt methods and techniques developed in [1, 2, 3] and [4] dealing with a semilinear elliptic equation.

Bibliography


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On a functional equation associated with a first order problem with finite delay

Carlos Lizama and Felipe Poblete*

Abstract

In this work, we study the functional equation

\[ G(s)(1 \ast G)(t) - (1 \ast G)(t) - (1 \ast FG)(s) = G(t)(1 \ast G)(s) - (1 \ast G)(s) - (1 \ast FG)(1 \ast G)(t), \]

for bounded operator valued functions \( G(t) \) defined on the positive real line \( \mathbb{R}_+ \). We show that, under some natural assumptions, the existence of solution of the functional equation characterizes the well posedness on \( C^1(\mathbb{R}_+; X) \), in a mild sense, the following problem of first order with finite delay on a Banach space \( X \)

\[
(I) := \begin{cases} 
  u'(t) = Au(t) + Fu_t & t > 0 \\
  u(0) = x \\
  u(t) = \phi(t) & -r < t < 0,
\end{cases}
\]

Functional equations arise in most parts of mathematics. Well known examples are Cauchy’s equation, the functional equations for the Riemann zeta function, the equation for entropy and numerous equations in combinatorics. Still other examples arise in probability theory, geometry and operator theory [1].

The theory of functional equations for bounded operators, emerged after the book of Hille and Phillips [2] in 1957. A strongly continuous semigroup \( T(t) \) of bounded and linear operators on a Banach space \( X \), is defined by means of Abel’s functional equation:

\[
\begin{align*}
  T(t)T(s) &= T(t+s), & t \geq 0, \\
  T(0) &= I,
\end{align*}
\]

which, in turn, characterizes the well posedness of the abstract Cauchy problem of first order:

\[
\begin{cases} 
  u'(t) = Au(t), & t \geq 0; \\
  u(0) = u_0,
\end{cases}
\]

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where \( Ax = \lim_{t \to 0^+} \frac{T(t)x - x}{t} \) is defined on the domain \( D(A) := \{ x \in X : \lim_{t \to 0^+} \frac{T(t)x - x}{t} \text{ exists in } X \} \). In 1966, Sova [5] introduces the concept of strongly continuous cosine operator functions, \( C(t) \), by means of D’Alembert’s functional equation:

\[
\begin{align*}
C(t + s) + C(t - s) &= 2C(t)C(s), \quad t, s \in \mathbb{R}; \\
C(0) &= I.
\end{align*}
\]

which characterizes the well posedness of the abstract Cauchy problem of second order:

\[
\begin{align*}
\left\{ \begin{array}{l}
\dddot{u}(t) = Au(t), \quad t \geq 0; \\
u(0) = u_0; \\
\dot{u}(0) = u_1.
\end{array} \right.
\end{align*}
\]

where now \( Ax = 2 \lim_{t \to 0^+} \frac{C(t)x - x}{t^2} \) is defined on \( D(A) := \{ x \in X : \lim_{t \to 0^+} \frac{C(t)x - x}{t^2} \text{ exists in } X \} \).

Let \( A \) be a linear operator defined on a Banach space \( X \). In [4] Prüss proved that the Volterra equation of scalar type:

\[
u(t) = \int_{0}^{t} a(t-s)Au(s)ds + f(t), \tag{0.0.13}\]

is well posed if and only if it admits a resolvent family, i.e. a strongly continuous family \( S(t) \) of bounded and linear operators which commutes with \( A \) and satisfies the so called resolvent equation [4, Definition 1.3]:

\[
S(t)x = x + \int_{0}^{t} a(t-s)AS(s)xds, \quad t \geq 0, \quad x \in X.
\]

Resolvent families of operators have been known for a long time. They have many applications in the study of abstract differential and integral equations. We note that choosing the kernels \( a(t) \equiv 1 \) and \( a(t) = t \) corresponds to the above mentioned cases of strongly continuous semigroups and cosine operator functions, respectively.

Recently, the authors in [3] studied a commutative and one parameter family of strongly continuous operators \( R_{a,k}(t) \), depending on two scalar kernels \( a(t) \) and \( k(t) \), satisfying \( R_{a,k}(0) = k(0)I \) and the functional equation

\[
R_{a,k}(s)(a*R_{a,k})(t) - (a*R_{a,k})(s)R_{a,k}(t) = k(s)(a*R_{a,k})(t) - k(t)(a*R_{a,k})(s), \quad t, s \geq 0. \tag{0.0.14}
\]

In case \( k(t) \equiv 1 \) and \( a(t) \) positive, one of their main results in [3] show that the functional equation (0.0.14) characterizes a resolvent family, and therefore the well-posedness of the Volterra equation (0.0.16). Moreover, the representation of the generator is given by

\[
Ax = \lim_{t \to 0^+} \frac{R_{a,k}(t)x - x}{\int_{0}^{t} a(s)ds}, \tag{0.0.15}
\]
for all \( x \in D(A) := \{ x \in X : \lim_{t \to 0^+} \frac{R_{a,1}(t)x - x}{(1 + a)(t)} \text{ exists in } X \} \), which includes the case of semigroups, cosine operator functions and resolvent families for \( a(t) = g(t), \alpha > 0 \).

Following the above ideas, in this work we will connect the existence of a commuting of strongly continuous family of operators \( \{ G(t) \}_{t \geq 0} \) defined by 0 for \(-r \leq t < 0\) which satisfies the functional equation (FE)

\[
G(s)(1 * G)(t) - (1 * G)(t) - (1 * FG_t)(1 * G)(s) = G(t)(1 * G)(s) - (1 * G)(s) - (1 * FG_s)(1 * G)(t)
\]

for all \( s, t \geq 0 \). With the well posedness on \( C^1(\mathbb{R}^+; X) \), in a mild sense, for the following problem of first order with finite delay on a Banach space \( X \)

\[
(I) := \begin{cases} 
  u'(t) = Au(t) + Fu_t & t > 0 \\
  u(0) = x \\
  u(t) = \phi(t) & -r < t < 0,
\end{cases}
\]

where the initial conditions \( x \in D(A) \) and \( \phi \in C([-r, 0], D(A)) \). Here, \( A \) is a closed operator with domain \( D(A) \subseteq X \) and \( F \) is a bounded linear map defined on an appropriate space.

To conclude the above mentioned, we motivate by using the Laplace transform, one definition of a retarded resolvent family \( \{ G(t) \}_{t \geq 0} \) and their relation with the well posedness of the problem \( (I) \) in a mild sense. More precisely, we say that a strongly continuous family of linear and bounded operators \( \{ G(t) \}_{t \geq 0} \), defined by 0 for \(-r \leq t < 0\), is a retarded resolvent with delay \( F \) if the following properties hold:

(i) \( G(0) = I \);

(ii) \( G(t)x \in D(A) \) and \( G(t)Ax = AG(t)x \) for all \( x \in D(A) \) and \( t \geq 0 \);

(iii) \( G(t)x = x + \int_0^t AG(s)xds + \int_0^t FG_s xds, \ t \geq 0, x \in D(A). \)

In such case we called \( A \) the generator of the retarded resolvent family \( \{ G(t) \}_{t \geq 0} \) with delay \( F \). Finally, we will give sufficient and necessary conditions on the retarded resolvent family \( \{ G(t) \}_{t \geq 0} \) to ensure that (FE) is satisfied. Here

\[
D(A) := \left\{ x \in X : \lim_{t \to 0^+} \frac{G(t)x - x - (1 * FG_t)x}{t} \text{ exists} \right\}
\]

and

\[
Ax := \lim_{t \to 0^+} \frac{G(t)x - x - (1 * FG_t)x}{t} \quad x \in D(A).
\]

**Bibliography**


Hölder continuous solutions for a fractional differential equations

Rodrigo Ponce

Abstract

We study the existence and uniqueness of solutions of an abstract fractional differential equation in Hölder spaces.

Using some results of Arendt, Batty and Bu [1], we study the existence and uniqueness of Hölder continuous solutions to equation

$$D^\beta u(t) = Au(t) + f(t), \quad t \in \mathbb{R},$$

(0.0.16)

where $A$ is a closed linear operator defined on a Banach space $X$, $f \in C^\alpha(\mathbb{R}; X)$, $0 < \alpha < 1$, and the fractional derivative for $\beta > 0$ is taken in the sense of Caputo. Existence of Hölder continuous solutions to fractional differential equations in the form of (0.0.16) have been studied for example, by Clement, Gripenberg and Londen using the method of the sum of Da Prato and Grisvard [2].

For $\beta > 0$, let $C^{\alpha,\beta}(\mathbb{R}, X)$ be the Banach space of all $u \in C^\alpha(\mathbb{R}, X)$, $n = \lfloor \beta \rfloor$, such that $D^\beta u$ exists and belongs to $C^{\alpha}(\mathbb{R}, X)$ equipped with the norm

$$\|u\|_{C^{\alpha,\beta}} = \|D^\beta u\|_{C^\alpha} + \sum_{j=1}^n \|D^{\beta-j} u(0)\|.$$  

Definition 0.6 We say that the equation (0.0.16) is $C^{\alpha}$-well posed if, for each $f \in C^{\alpha}(\mathbb{R}; X)$, there exists a unique function $u \in C^\alpha(\mathbb{R}; [D(A)]) \cap C^{\alpha,\beta}(\mathbb{R}; X)$, and the equation (0.0.16) holds for all $t \in \mathbb{R}$.

The following Theorem is the main result of this talk. Its prove is based in the theory of $C^{\alpha}$-multipliers introduced in [1].

Theorem 0.7 Let $A : D(A) \subseteq X \to X$, be a linear closed operator defined on Banach space $X$. Then, the following assertions are equivalent

(i) The equation (0.0.16) is $C^{\alpha}$-well posed;

(ii) $(i\eta)^\beta \in \rho(A)$ for all $\eta \in \mathbb{R}$ and $\sup_{\eta \in \mathbb{R}} \|(i\eta)^\beta ((i\eta)^\beta - A)^{-1}\| < \infty$.

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Applying the results in [4], we study the existence of Hölder continuous solutions to problem

\[
\begin{aligned}
D^\beta u(t, x) &= \Delta u(t, x) + f(t, x), & t \in \mathbb{R}, \\
 u &= 0 & \text{in } \mathbb{R} \times \partial \Omega,
\end{aligned}
\]  

(0.0.17)

where \(0 < \beta < 1\), \(\Omega\) is a bounded domain in \(\mathbb{R}^n\) with a smooth boundary \(\partial \Omega\), introduced in physics by Nigmatullin [3] to describe diffusion in special types of porous media.

Bibliography


Strong solutions of abstract fractional differential equations

Juan C. Pozo*, Hernán R. Henríquez and Verónica Poblete

Abstract

In this work we establish the existence of strong solutions for abstract semi-linear fractional differential equations. We consider the autonomous and non-autonomous case. Our approach relies on the existence of a resolvent family with bounded semivariation for the homogeneous autonomous equation, and appropriate conditions on the forcing function.

In this work we study the existence of strong solution of abstract Cauchy problem of fractional order. Let $X$ be a Banach space and suppose that $A(t) : D(A(t)) \subseteq X \to X$ are closed linear operators with domain $D(A(t)) = D$ for all $t \in [0, a]$, $a > 0$. We consider the following fractional differential equation

\[
D_\alpha^t u(t) = A(t)u(t) + f(t, u(t)), \quad t \in [0, a],
\]

\[
\begin{aligned}
&u(0) = x, \\
u'(0) = y.
\end{aligned}
\]

where $\alpha \in (1, 2)$, and the fractional derivative $D_\alpha^t$ is understood in the Caputo sense.

If $A(t) = A$ for all $t \in [0, a]$, the problem (0.0.18) is known in the literature by fractional abstract Cauchy problem associated to $A$ of order $\alpha$. The existence of solutions of this problem is strongly related with the concept of $\alpha$-resolvent family $\{S_\alpha(t)\}_{t \geq 0}$, developed by Bazhleкова [2]. In fact, the fractional differential equation (0.0.18) is well posed if and only if $A$ is the infinitesimal generator of an $\alpha$-resolvent family $\{S_\alpha(t)\}_{t \geq 0}$. Assuming that $A$ is the infinitesimal generator of an $\alpha$-resolvent family $\{S_\alpha(t)\}_{t \geq 0}$, is well known that a strong solution of the problem (0.0.18) verifies the formula

\[
u(t) = S_\alpha(t)x + (g_1 * S_\alpha)(t)y + (g_{\alpha-1} * S_\alpha * f)(t), \quad t \in [0, a],
\]

However, a continuous function $u : [0, a] \to X$ described by the preceding formula is not necessarily a strong solution of problem (0.0.18). This fact motivates the introduction of a weaker concept of solution of the abstract fractional Cauchy problem. A continuous function $u : [0, a] \to X$ is called mild solution of the abstract fractional differential Cauchy problem if $u$ satisfies the formula (0.0.19).

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The existence of strong solutions of the problem (0.0.18) when $f \in C([0, a]; X)$ has been analyzed by F. Li and M. Li ([5]). Specifically, they show that for all $f \in C([0, a]; X)$ the corresponding mild solution is a strong solution of the problem (0.0.18) if and only if the $\alpha$-resolvent family has bounded semivariation in the interval $[0, a]$. This result can also be derived as a particular case of the theory developed by H. Thieme [6]. However, in both works, the authors do not exhibit an explicit example of an unbounded operator $A$ that generates an $\alpha$-resolvent family of bounded semivariation. This subject was originally studied for the abstract Cauchy problem of first order in [1, 4, 7]. These authors establish that there are $C_0$-semigroups of bounded semivariation and generated by unbounded operators. Later, this same topic for the abstract Cauchy problem of second order was studied in [3]. In this case, the authors establish that a cosine function with bounded semivariation can only be generated by a bounded operator. It remains open to decide if there exist $\alpha$-resolvent families of bounded semivariation for $1 < \alpha < 2$. In our main results we construct a concrete example of an $\alpha$-resolvent family of bounded semivariation where its infinitesimal generator is an unbounded operator. Moreover, we will establish the existence of strong solutions of problem (0.0.18) if $f$ is a $X$-valued continuous function on $[0, a]$. The results are based on the properties of $\alpha$-resolvent families of bounded semivariation and appropriate conditions on $f$. We apply our results to study semilinear and non-autonomous fractional differential equations.

Bibliography


Existence of solutions for a fractional neutral integro-differential equation with unbounded delay

José Paulo Carvalho dos Santos

Abstract

In this talk, we study the existence of mild solutions for the neutral fractional integral evolutionary equation

\[ D_t^\alpha (x(t) + f(t, x_t)) = Ax(t) + \int_0^t B(t-s)x(s)ds + g(t, x_t), \quad t > 0, \]
\[ x_0 = \varphi, \quad x'(0) = 0, \]

where \( \alpha \in (1, 2); A, (B(t))_{t \geq 0} \) are closed linear operators defined on a common domain which is dense in a Banach space \( X \), \( D_t^\alpha h(t) \) represent the Caputo derivative of \( \alpha > 0 \) defined by

\[ D_t^\alpha h(t) := \int_0^t g_{n-\alpha}(t-s) \frac{d^n}{ds^n} h(s)ds, \]

where \( n \) is the smallest integer greater than or equal to \( \alpha \) and \( g_{\beta}(t) := \frac{t^{\beta-1}}{\Gamma(\beta)}, t > 0, \beta \geq 0 \). The history \( x_t : (-\infty, 0] \to X \) given by \( x_t(\theta) = x(t+\theta) \) belongs to some abstract phase space \( B \) defined axiomatically and \( f, g : I \times B \to X \) are appropriate functions. This talk is based on work with Bruno de Andrade.

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On evolutionary differential equations with state-dependent delay

Giovana Siracusa* and Bruno de Andrade

Abstract

The study of the topological structure of solution set of differential equations dates back to the beginning of the 20’s when H. Kneser (see [7]) proved that the Peano existence theorem could be reformulated to ensure that the solution set of a ODE is, beyond nonempty, a compact and connected set. This property is known in the literature as the Kneser’s property. Almost 20 years later, N. Aronszajn (see [4]) improved the Knenser theorem showing that the set of all solutions of a ODE is an $R^3$-set, that is, an intersection of a decreasing sequence of compact absolute retracts sets. Evidently the Aronszajn theorem had a large impact on qualitative theory of differential equations and due to this the study of topological structure of the solution set of differential equations has drawn attention of researchers in the last years (see for instance [1, 2, 3, 5, 6, 8] and references therein).

In this work we study some topological properties of the solution set of differential equation with state-dependent delay

\[
\begin{align*}
& u'(t) = Au(t) + f(t, u_{\rho(t,u)}), \quad t \in [0, b], \\
& u_0 = \varphi \in \mathcal{B},
\end{align*}
\]

(0.0.20)

where $A : D(A) \subset X \to X$ is infinitesimal generator of the semigroup of linear operators $\{S(t); t > 0\}$ on a Banach space $X$ and the history $x_t : (-\infty, 0] \to X$, given by

\[x_t(\theta) = x(t + \theta),\]

belongs to phase space $\mathcal{B}$ described axiomatically. The functions $f : [0, b] \times \mathcal{B} \to X$ and $\rho : [0, b] \times \mathcal{B} \to (-\infty, b]$ are given functions.

Bibliography


On a class of discontinuous dynamical systems

Miguel V. S. Frasson, Marta C. Gadotti, Selma H. J. Nicola and Plácido Z. Táboas*

Abstract

The object of study are the so called impulsive differential equations, where the involved equation is autonomous and the impulses, previously unknown, are given by intrinsic causes. Therefore the whole system is autonomous and define a discontinuous semi-group. We give some examples showing peculiarities of these systems and that, even when the involved equations are simple linear equations, they can exhibit interesting dynamics. We present a topological approach to deal with this kind of problem.
Stability results for measure neutral functional differential equations via GODE

Márcia Federson and Patricia H. Tacuri*

Abstract

We consider a class of measure neutral functional differential equations whose integral form is given by

\[ x(t) - x(0) = \int_0^t f(x(s), s) ds + \int_{-\infty}^0 d\Phi[\mu(t, \theta)] x(t + \theta) - \int_{-\infty}^0 d\Phi[\mu(0, \theta)] \varphi(\theta) \]

and we establish stability results using the correspondence between solutions of this equation and solutions of a generalized ordinary differential equations. We introduce the concept of regular stability of linear operators on a Banach space of \( \mathbb{R}^n \)-valued regulated functions. We discuss the total stability for a class of measure neutral functional differential equations.

Bibliography


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Supported by FAPESP grant 2012/18559-1. Department of Mathematics and Computer Sciences, São Paulo State University “Júlio Mesquita Filho”, Presidente Prudente, Brazil, e-mail: ptacuri@fct.unesp.br
On a class of thermoelastic plates with $p$-Laplacian

To Fu Ma

Abstract

In recent years the class of vibrating plates with $p$-Laplacian
\[ u_{ttt} + \Delta^2 u - \Delta_p u = \{\text{damping and forcing}\}, \]
defined on bounded domains of $\mathbb{R}^n$, was studied by several authors (see for instance [1,2,3,4] and the references therein). The present paper contains a first thermoelastic model of this class of problems including both Fourier and non-Fourier heat laws. We discuss the modeling and the well-posedness of the problem.

Bibliography


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Asymptotically almost automorphic and almost automorphic solutions of Volterra Integral Equations on time scales

Eduard Toon

Abstract

This is a joint work with Carlos Lizama, Jaqueline G. Mesquita and Rodrigo Ponce. In this work, we investigate the existence and uniqueness of almost automorphic solutions of semilinear Volterra Integral Equation on time scales given by:

\[ u(t) = \int_{-\infty}^{t} a(t, \sigma(s))[u(s) + f(s, u(s))] \Delta s, \]

where \( a : \mathbb{T} \times \mathbb{T} \to \mathbb{R}^{n \times n} \) is almost automorphic in both variables and \( f : \mathbb{T} \times \mathbb{R}^{n} \to \mathbb{R}^{n} \).

We also show a characterization of asymptotically almost automorphic solution of the following semilinear Volterra Integral Equation:

\[ u(t) = \int_{t_0}^{t} a(t, \sigma(s))[u(s) + f(s, u(s))] \Delta s, \]

with \( t_0 \in \mathbb{T}_+, t > t_0 \), where \( a : \mathbb{T}_+ \times \mathbb{T}_+ \to \mathbb{R}^{n \times n} \) is almost automorphic on both variables and \( f : \mathbb{T}_+ \times \mathbb{R}^{n} \to \mathbb{R}^{n} \).

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Soluciones Convergentes en Ecuaciones Diferenciales Impulsivas con Avance

Manuel Pinto and Ricardo Torres N.*

Abstract

Las ecuaciones diferenciales con argumento constante a trozos (DEPCAG) tienen soluciones continuas y en los extremos de los intervalos de constancia generan una ley recursiva, una ecuación discreta, ver [9, 6, 7]. Al tener en estos puntos un salto, aparecen las ecuaciones diferenciales con argumento constante a trozos (IDEPCAG) con impulsos. Este tipo de ecuaciones corresponden al tipo híbridas, ya que combinan propiedades tanto de ecuaciones discretas como de continuas. Ver [1, 2, 8, 9]

En esta ocasión mostraremos la existencia y unicidad de soluciones del sistema impulsivo con argumento constante a trozos

\[ X'(t) = A(t)X(t) + B(t)X(\gamma(t)) + F(t), \quad t \neq t_i \]
\[ \Delta X(t)_{t=t_i} = C_iX(t_i^-) + D_i, \quad t = t_i \]

ayudados del análisis de los sistemas

\[ X'(t) = A(t)X(t), \quad t \neq t_i \]
\[ \Delta X(t)_{t=t_i} = C_iX(t_i^-), \quad t = t_i \]

y

\[ X'(t) = A(t)X(t) + B(t)X(\gamma(t)), \quad t \neq t_i \]
\[ \Delta X(t)_{t=t_i} = C_iX(t_i^-) + D_i, \quad t = t_i \]

con \( A(t), B(t) \) y \( F(t) \) funciones continuas a trozos localmente integrables, \((t_i)_{i\in\mathbb{N}}\) partición del intervalo \([t_0, \infty)\) con \( t_i < t_{i+1}, \forall i \in \mathbb{N}, \) en donde se considera un avance discontinuo

\[ \gamma(t) = t_{i+1}, \quad \text{si} \ t \in [t_i, t_{i+1}], \forall i \in \mathbb{N}. \]

En nuestros resultados tiene un rol fundamental la invertibilidad de la matriz

\[ J(t, t_i) = I + \int_{t_i}^{t} \Phi(t, s)B(s)ds, \quad \forall t \in [t_i, t_{i+1}], \forall i \in \mathbb{Z} \]

*Partially supported by Fondecyt 1120709. Departamento de Matemática. Facultad de Ciencias, Universidad de Chile. Santiago, Chile, e-mail: ricardotorresn@gmail.com
con $\Phi$ matriz fundamental del sistema homogéneo asociado.

Las ecuaciones discretas asociadas juegan un gran papel, ya que son una aproximación a las soluciones de la ecuación a tiempo continuo. También son ecuaciones con avance y sus soluciones convergentes, ver [2, 3]. La ecuación diferencial es una aproximación de la ecuación diferencial ordinaria, en cuanto

$$\sigma = \sup_{i \in \mathbb{N}}|t_{i+1} - t_i|,$$

sea pequeño. Ver [4, 5]

Se obtiene la fórmula de variación de parámetros asociada, cuya matriz fundamental es particularmente especial.

Probamos que si los coeficientes son integrables, entonces las soluciones son convergentes.

**Bibliography**


Asymptotic behaviour of the time-fractional telegraph equation

Vicente Vergara

Abstract

We obtain the long-time behaviour to the variance of the distribution process associated with the solution of the telegraph equation. To this end, we use a version of the Karamata-Feller Tauberian theorem.

Bibliography


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